SUMO - Supermodeling by combining imperfect models

Workpackage 3: Year 2

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Author’s comments

Efforts in WP3 in the second year of the project were mostly pursuant to the Task 3.3 in the DOW.

Task 3.3 Develop learning strategies for intermediate complexity climate supermodels: The work will be further continued on analysis of quasi-geostrophic (QS) channel model, which is a relatively simple geophysical model. MASA investigators have previously worked on this model showing that a single connection coefficient in a supermodel consisting of two QS channel models could be adapted using the incremental learning approach. It had been shown in previous work that two such models, one with a forcing jet in the Atlantic, and another with a jet in the Pacific, could be made to synchronize when connected, each model inducing the missing jet in the other (Duane and Tribbia, 2001, 2004). This model configuration in these studies was originally researched as to predict new types of Atlantic-Pacific teleconnections. But it is also a supermodel. It can be trained by connecting to a third “real” channel with two jets and provides an ideal test case to further develop an incremental learning approach. The QS channel investigation will form the basis for the subsequent study of the more realistic quasi-geostrophic model, the Ecbilt model, that will be studied in WP4.

In addressing the issue of learning in the QS model, we were led to focus on the fundamental issue of the advantages of the QS supermodel as compared to simpler methods for combining individual QS models, as described in the following report.
1 Introduction: The old Atlantic-Pacific supermodel

A primitive supermodel was previously created by fusing two models of the mid-latitude flow, one with forcing in the Atlantic sector and the other with forcing in the Pacific sector, to form a two-sector model (Duane and Tribbia, 2001). While a two-sector model is of course easily formulated, the construction from single-sector models illustrates the utility of the synchronization paradigm and of supermodeling.

The uncoupled single-system model is given by the quasigeostrophic equation for potential vorticity $q$ in a two-layer reentrant channel on a $\beta$-plane

$$\frac{Dq_i}{Dt} = \frac{\partial q_i}{\partial t} + J(\psi_i, q_i) = F_i + D_i$$ (1)

where the layer $i=1, 2$, $\psi$ is streamfunction, and the Jacobian $J(\psi, \cdot) = \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x}$ gives the advective contribution to the Lagrangian derivative $D/Dt$ (Vautard et al., 1988; Vautard and Legras, 1988). The forcing $F$ is a relaxation term designed to induce a jet-like flow near the beginning of the channel:

$$F_i = \mu_0 (q_i^* - q_i)$$ (2)

for $q_i^*$ corresponding to a choice of streamfunction $\psi^*$ shown in Fig. 2a. The dissipation terms $D$, boundary conditions, and other parameter values are given in (Duane and Tribbia, 2004).

We couple two models of the form (1), with $F_i = \mu_0 (q_i^* - q_i)$, using a different $q_i^*$ in each model. The coupling here is in the advection terms of the two models. The configuration is given by

$$\frac{Dq^A}{Dt} + cJ(\psi^A, q^B - q^A) = F^A + D^A$$

$$\frac{Dq^B}{Dt} + cJ(\psi^B, q^A - q^B) = F^B + D^B$$ (3)

with forcing terms defined in terms of their spectral components

$$F^{A,B}_k = (\mu_0 - \mu^*_k) [q^{A,B*}_k - q^{A,B}_k]$$ (4)

where $q_k$ is the wave number $k$ spectral component of $q$ (suppressing the layer index $i$). The coefficients $\mu^*_k$ are chosen so as to couple only the medium scales as described in detail in (Duane and Tribbia, 2004). The flow fields in the coupled channels governed by (3) are found to synchronize, regardless of differences in initial conditions, as seen in Fig. 2. The correspondence is not exactly the identity, because of the difference in forcings, but is an instance of generalized synchronization. Likewise here, at $c = 1/2$ we have $\psi^A \approx \psi^B$.

It is readily seen that the average $\tilde{q} = (q^A + q^B)/2$ of the solutions of (3a) and (3b), for strong coupling $c = 1/2$, is the solution of a model with the average forcing, that is, of a model with $\tilde{q}^* \equiv (q^{A*} + q^{B*})/2$, describing two jets, as in Fig. 2g. The flow in either
Figure 1: Two quasigeostrophic channel models (Duane and Tribbia, 2004), one with a better representation of the Atlantic jet stream and the other with a better representation of the Pacific jet stream, were fused to form a supermodel by adapting a connection coefficient so as to reproduce “truth” in a training set. After \( n = 30000 \) time steps, the models were synchronized with each other and with truth. Contours represent streamlines of the flow, as in (Duane and Tribbia, 2004). (The domain actually consists of two channels in the top and bottom half of each panel, resp., with flows in opposite directions, a configuration that was introduced for convenience, in that periodic boundary conditions can be imposed in the \( y \)-dimension.)
Figure 2: The forcing $\psi^*$ (a,b), and the evolving flow $\psi$ (c-f), in the parallel channel model with advective coupling (given by (3)), with $c = 1/2$, and model parameters as in (Duane and Tribbia , 2004), for the indicated numbers $n$ of time steps in a numerical integration. One time step is $\approx 0.4$ hr. Near-identical synchronization occurs by the last time step shown (e,f). The synchronized flows closely approximate the flow in a model with forcing $\hat{\psi}^*$ (corresponding to $\hat{q}^* \equiv (q^A + q^B)/2$) describing jets in both sectors (g). (Boxed regions were used in (Duane and Tribbia , 2004) to study the zonal/blocked index cycle.)
channel $\psi^A \approx \psi^B$ approximates the flow $\hat{\psi}$ in a channel with two jets. As the coupling between the channels is increased from $c = 0$ to $c = 1/2$, the dynamics of each channel changes so as to incorporate an approximate, “virtual” counterpart of the dynamics of the sector that is forced in the other channel.

The desired value $c = 1/2$ can be obtained by adapting to a real (two-jet) data set $q_{obs}(x, t)$ according to

$$\frac{dc}{dt} = a \int dx [J(\psi^A, q^B - q^A)(q^A - q_{obs}) + J(\psi^B, q^A - q^B)(q^B - q_{obs})] \quad (5)$$

as was indeed tested, with results shown in Fig. 3. As the coupling adapts, we obtain 3-way synchronization among “truth”, the Atlantic-jet model, and the Pacific-jet model, as shown in Fig. 1. The slowness of the convergence and the instability of the resulting state suggest the need for an improved learning algorithm, although the synchronization-based algorithm is simple and versatile.

It is expected that greater stability would result by defining a cost over an extended window in time, generalizing the instantaneous synchronization error. That approach would coincide with learning algorithms previously applied successfully to create a Lorenz ’63 supermodel (van den Berge et al., 2010; Mirchev et al., 2012). The trade-off is with the greater computational cost of the finite-time-cost algorithms, an issue that is presently being studied.

1.1 The Atlantic-Pacific QG configuration as a test of the utility of supermodeling

To investigate the utility of supermodeling, one may compare the supermodel formed from models with single-sector forcing with averages of the outputs of the single sector models. The results would appear disappointing. Averaging of the two models gives blocked-zonal vacillation in each sector, just as in the supermodel, with similar blocking.
frequencies. The supermodel would appear only to capture details of the relationship between the two sectors - anticorrelation between blocking activity in the two sectors (see (Duane and Tribbia, 2004)), but that relationship is weak and of little practical interest.

More generally, the nonlinearities of the Lorenz systems on which supermodeling was first demonstrated seem less relevant - one is interested in mean quantities or in statistical properties that do not require detailed reconstruction of the true attractor of the high-dimensional system. If those properties vary linearly as parameters are varied, there is no advantage in supermodeling as compared to ex post facto averaging. It is therefore suggested that the QG model provides a useful arena in which to explore the advantages of supermodeling, and to determine conditions under which nonlinearities render it superior to output averaging.

2 Nonlinearities in PDE systems of Atmospheric Relevance: Bifurcations?

As a thought experiment, imagine that the dynamical parameters of the true system, or the best low-resolution model of the true system, are close to some bifurcation point, on the other side of which the dynamics change qualitatively. Now imagine that one forms a supermodel from a collection of individual models, some of which are in this qualitatively unphysical region of parameter space. Then it is easy to see how a supermodel would likely be superior to any average of outputs: A weighted supermodel could reproduce the true parameters by choosing appropriate weights for the tendencies, provided the numbers of free parameters in those tendencies are not too large as compared to the number of models. The supermodel’s prediction would consist of high-dimensional fields generated using those correct parameter values. A connected supermodel would arguably behave similarly, with even more flexibility due to the larger number of free connection coefficients. An output average, on the other hand, would include some fraction of the unphysical fields in the prediction. It seems unlikely that even a judicious choice of ex post facto weights would generally be enough to cause cancellation of the unphysical components of the behavior of the high-dimensional fields. (One may be interested only in various mean properties of these fields, but with increasing interest in detailed regional projections, the problem becomes closer to that of reproducing the full fields.)

Thus it would seem that supermodels would be most advantageous in regions of parameter space separated by bifurcations, with individual models on either side. The question is then whether such a situation arises in collections of models that are actually used. The usual answer is that it does not. For the related issue of bifurcations that might be caused by an increase in greenhouse gas (GHG) levels, one can consider for instance the data in Fig. 4, describing changes in August temperature, evaporation, soil moisture, and net solar radiation in the Netherlands in a 17-member ensemble of ECHAM5 as GHG’s increases.

The changes are smooth and nearly linear, with no obvious bifurcations. It is commonly thought that parameter variations linking the different models will produce sim-
Figure 4: Monthly mean time series for August in the Netherlands for surface air temperature (upper left), evaporation (upper right), soil moisture (lower left), and net surface solar radiation (lower right), in an ECHAM5 ensemble run. Red crosses denote the 17 individual ensemble members, the black line is the ensemble mean, the blue line connects the values of ensemble member 13.
ilarly smooth changes. Models do differ qualitatively in their representations of sub-
gridscale processes, but the effects are often represented as noise, and may not yield
sharp changes in the behavior on larger scales. In any case, the dynamics of the QG
models considered presently are too simple to allow a study of such effects. We therefore
look elsewhere to find an advantage in supermodeling.

3 Other Nonlinearities: Nonmonotonic response to forcing

Without bifurcations, simple nonlinearity can still make the supermodel superior to an
average of model outputs. This is perhaps most easily seen in the case where diagno-
sic properties depend non-monotonically on system parameters. Suppose we have two
models of the form:

\[ \begin{align*}
\dot{x} &= F(x, p_1) \\
\dot{x} &= F(x, p_2)
\end{align*} \]  

where \( F \) is linear in the parameter \( p \), and consider some diagnostic \( P(p) \), e.g. mean
temperature. Further suppose that \( P(p_1) = P(p_2) \), but that for some intermediate
value \( p_i, p_1 < p_i < p_2, P(p_i) > P(p_1) \). Then any weighted average of model outputs
will only give the first value \( P(p_1) \). A weighted supermodel, on the other hand, could
readily reproduce the correct dynamics, that is \( F(x, p_i) = w_1 F(x, p_1) + w_2 F(x, p_2) \) for
appropriately chosen weights \( w_1 \) and \( w_2 \). It is hypothesized that a connected supermodel
could also give the correct result.

The QG channel model described above vacillates between two dynamical regimes
described as "blocked" and "zonal", with examples given in Fig. 5. The response of
the blocking activity to the forcing parameter \( \mu_0 \) in (2) provides a simple example of
non-monotonic behavior. For zero forcing, blocking frequency is zero due to damping
by the dissipative terms. For large forcing, the flow is consistently jet-like, and again
there is no blocking. Typical flow fields for these two cases are shown in Fig. 6. (The
zero-forcing flow in Fig. 6a is turbulent, but of low amplitude, and includes no blocks.)

3.1 Weighted supermodel

A weighted supermodel formed from the two individual models illustrated in Fig. 6 can
reproduce the true dynamics exactly for any value of the forcing coefficient \( \mu_0 \) between
\( \mu_0 = 0 \) and \( \mu_0 = 3 \). For the typical value \( \mu_0 = 0.3 \) used previously, the behavior is as
illustrated in Fig. 7. The supermodel flow spends much time in the blocked regime,
unlike the flows in the individual models or any weighted average thereof. (If the actual
flow fields of the individual models are averaged, instead of the blocking frequencies, the
same conclusion is reached.)

The learning task for the weights is equivalent to that for determining the single
parameter \( \mu_0 \) directly. A previous algorithm for parameter learning in models that
synchronize with identical parameters (Duane et al., 2007), for instance, is effective in
Figure 5: Streamfunction (in dimensional units of $1.48 \times 10^9 m^2 s^{-1}$) describing the forcing $\psi^*$ (a), a typical zonal flow state (b), and a typical blocked flow state (c) in the two-layer quasigeostrophic channel model. Parameter values are as in (Duane and Tribbia, 2004). An average streamfunction for the two vertical layers $i = 1, 2$ is shown.
Figure 6: Typical flows in the QG channel model with very small forcing coefficient ($\mu_0 = 0$) (a), and very large forcing coefficient ($\mu_0 = 3.0$) (b). (The domain in each panel includes two channels with flows in opposite directions, as in Fig. 1).

Figure 7: Typical flow in the QG channel model with a “realistic” forcing coefficient ($\mu_0 = 0.3$) (a), and the history of vacillation of the flow in the bottom half of the domain between zonal and blocked regimes, sampled at low temporal resolution over the course of a simulation (b), using the blocking diagnostic defined in (Duane and Tribbia, 2004).
Figure 8: The evolving flow $\psi$ (a-f) for two quasigeostrophic channel models coupled through mid-range spectral components as in (Duane and Tribbia, 2001, 2004) (but in one direction only), and with the forcing parameter $f^B \equiv \mu_0$ for the second channel adapted on the fly according to the learning rule in (Duane et al., 2007). Starting from the initial value $f^B = 3.0$ at time step $n = 0$ (not shown), $f^B$ converges (g) to the value of the corresponding parameter $f^A = 0.3$ (dashed line) in the first channel, as the flows synchronize. (An average of the two layers $l = 1, 2$ is shown.)
the present context. Results for training with a set of mid-range spectral coefficients of
the “true” flow are reproduced in Fig. 8.

The synchronization-based algorithm adjusts weights “on the fly” as with the Atlantic-
Pacific supermodel described above. But as in that case, learning algorithms with costs
defined over finite temporal windows could also be prescribed.

It is hypothesized that a connected supermodel could also be formed from the two
individual models illustrated in Fig. 6 that would approximate the “true” behavior for
arbitrary forcing coefficient. Details remain to be worked out.

4 Forcings in IPCC climate-change scenario

While a supermodel is clearly superior to an output average in the example given above,
and in extreme cases generally, more linear behavior is expected for smaller inter-model
differences as might occur in a realistic suite of models, such as the IPCC set. To
construct a realistic experiment with toy models, a correspondence is needed between
parameter differences among the toy models on the one hand, and differences among
models or parameters used in actual climate projection on the other. Considering the
forcing coefficient \( \mu \) in the QG models as representative of forcing generally, the question
is about the relative magnitude of differences in forcing among the different models.
External forcing in the different models is about the same, but the effective forcing,
when differences in internal dynamics are included, varies significantly. The differences
are manifest as differences in climate sensitivity - mean temperature change for given
increase in greenhouse gas levels. Sensitivities of the climate models in the IPCC suite
are listed in Fig. 9. They are seen to vary by about \( \pm 1/3 \) of the average value. We take
the QG forcing coefficient as analogous to these sensitivities, and introduce differences
between the coefficients in the different models of the same relative magnitude. So we
use models with \( \mu_0 = 0.2 \) and \( \mu_0 = 0.4 \) to form a supermodel.

Typical flow fields for the three values of the forcing coefficient \( \mu_0 = 0.2, 0.3, 0.4 \) are
shown in Fig. 10, along with blocked/zonal vacillation behavior. Unlike the case dis-
cussed above, it appears that if the two individual models err in their forcing coefficients
only to a degree that seems realistic, a weighted average of their blocking frequencies
could reproduce the “true” behavior. At least in regard to blocking frequency, the ad-
vantage of supermodeling is lost in this less extreme case.

5 Software framework for coupling PDE’s and climate models

In the realm of tool development, the Data Assimilation Research Testbed (DART) at the
National Center for Atmospheric Research (NCAR) has now been modified, according
to the plan described in the Year 1 report, so that a suite of models can assimilate data
from each other as well as from “truth”, effectively forming a connected supermodel.
Preliminary results with Lorenz systems, with fixed connections, are shown in Fig. 11.
DART is constructed so that a state vector for an arbitrary model can be modified based
on observed input, and then used to re-initialize the model. Constructing a supermodel in
Figure 9: Equilibrium climate sensitivities and transient climate responses (both in °C) for the various climate models used by the Intergovernmental Panel on Climate Change (IPCC) (IPCC, 2007).

<table>
<thead>
<tr>
<th>AOGCM</th>
<th>Equilibrium climate sensitivity (°C)</th>
<th>Transient climate response (°C)</th>
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<tbody>
<tr>
<td>1: BCC-CM1</td>
<td>n.a.</td>
<td>n.a.</td>
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<tr>
<td>2: BCCR-BCM2.0</td>
<td>n.a.</td>
<td>n.a.</td>
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<tr>
<td>3: CCSM3</td>
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<td>1.5</td>
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<tr>
<td>4: CGCM3.1(T47)</td>
<td>3.4</td>
<td>1.9</td>
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<tr>
<td>5: CGCM3.1(T63)</td>
<td>3.4</td>
<td>n.a.</td>
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<tr>
<td>6: CNRM-CM3</td>
<td>n.a.</td>
<td>1.6</td>
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<tr>
<td>7: CSIRO-MK3.0</td>
<td>3.1</td>
<td>1.4</td>
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<tr>
<td>8: ECHAM5/MPI-OM</td>
<td>3.4</td>
<td>2.2</td>
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<tr>
<td>9: ECHO-G</td>
<td>3.2</td>
<td>1.7</td>
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<td>10: FGOALS-g1.0</td>
<td>2.3</td>
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<td>11: GFDL-CM2.0</td>
<td>2.9</td>
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<td>12: GFDL-CM2.1</td>
<td>3.4</td>
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<td>23: UKMO-HadGEM1</td>
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Figure 10: Typical flows in the QG channel model for realistically low forcing $\mu_0 = 0.2$ (a), the “true” value $\mu_0 = 0.3$ (c), and realistically high forcing $\mu_0 = 0.4$ (e), with corresponding blocked/zonal vacillation histories (b), (d), (f).
the same manner will allow general connection and learning algorithms to be transferred between models, almost transparently to the user. Most of the effort was in creating inter-model assimilation software in a fully general manner, so that much less time will be needed to connect large climate models.

6 Summary and conclusions

The advantage of supermodeling over ex post facto averaging of outputs has been evaluated in regard to the index cycle in a simple quasigeostrophic channel model. Non-monotonic behavior as the forcing coefficient varies gives the supermodel a clear advantage if the individual models are defined with very large and very small forcings. (It appears that the weights or connections defining the supermodel could be readily learned from training data using methods previously articulated.) The advantage, however, is lost if the magnitude of the parameter differences among the individual models is arguably realistic.
It is expected that the general conclusion will carry over to configurations of QG models created by varying other parameters, e.g. static stability, provided that the models are not close to bifurcations. Of course, the detailed attractor is reproduced better by the supermodel, and higher-order statistics, like the East-West covariances described in the first example, may be qualitatively improved. The overall conclusion is that the advantage of supermodeling is to be sought in situations where there are structural differences between the individual models, e.g. in the parametrizations of subgrid-scale processes (not realizable in the simple QG example), provided that they result in qualitative differences in large-scale quantities of interest.
References


