Synchronicity From Synchronized Chaos

Gregory S. Duane*

Dept. of Atmospheric and Oceanic Sciences
UCB 311
University of Colorado
Boulder, CO 80309-0311

and

Macedonian Academy of Sciences and Arts
Bul. Krste Misirkov 2, P.O. Box 428
1000 Skopje, Republic of Macedonia

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* corresponding author: email address gregory.duane@colorado.edu
Abstract

The synchronization of loosely coupled chaotic oscillators, a phenomenon investigated intensively for two decades, may realize the philosophical notion of “synchronicity”. Effectively unpredictable chaotic systems, coupled through only a few variables, commonly exhibit a predictable relationship that can be highly intermittent, as with philosophical “synchronicities”. Synchronicity between matter and mind is realized dynamically if mind is analogized to a computer model assimilating observed data, as in meteorology. Synchronicities are meaningful, as philosophically required, if meaningfulness is related to internal coherence. Internal synchronization indeed appears necessary for synchronization between systems, as illustrated for a pair of forced-dissipative systems and for a pair of Hamiltonian systems that exhibit coherent structures. Synchronicity-as-synchronized-chaos has implications both for neural systems (biological and artificial) and for basic physics. In the former realm, it may be useful to dynamically induce different computational models of the same objective process to synchronize with one another as well as with that process - as may occur also in conscious mental processing. Basic physical synchronicity is manifest in the non-local quantum connections implied by Bell’s theorem. The quantum world resides on a generalized synchronization “manifold”, that one can either take as primitive or as evidence of a multiply-connected spacetime.

**Keywords:** synchronized chaos, on-off intermittency, machine perception, coherent structures, quantum nonlocality, microwormholes
I. INTRODUCTION

Synchronization within networks of oscillators is widespread in nature, even where the mechanisms connecting the oscillators are not immediately apparent. One recalls the example of the synchronization of clocks suspended on a common rigid wall, a paradigm commonly attributed to Huygens. As with similar phenomena of fireflies blinking in unison, or female roommates synchronizing hormonal cycles, the pattern suggests a universally valid organizational principle that transcends any detailed causal explanation. Further from everyday experience, but perhaps related [16], are the quantum mechanical harmonies between distant parts of a system that are not causally connected, involving also the observer’s choice of measurements as implied by Bell’s Theorem [6].

The study of coupled networks of oscillators in physical systems has focussed on regular oscillators with periodic limit-cycle attractors. Such models afford explanations for such surprising relationships as the one observed by Huygens, but other synchronous relationships that are sometimes said to exist in nature are less easily explained. While the synchronization of chaotic oscillators with strange attractors has become familiar in the last two decades, most work on such systems has examined engineered systems, primarily for application to secure communications, using the low-dimensional signal connecting the oscillators as a carrier that is difficult to distinguish from noise. However, examples of synchronized chaos in pairs of systems of partial differential equations that describe physical systems, coupled loosely, have also been given [11, 12, 14, 15, 32].

Synchronous relationships that are difficult to explain causally have figured prominently in primitive cultures and certain philosophical traditions [42]. The notion of “synchronicity” commonly associated with Jung$^1$ has two essential characteristics beyond the simple simultaneous occurrence of corresponding events: First, the simultaneous occurrences or “synchronicities” must be isolated occurrences. Second, the synchronicities must be “meaningful”. The idea of synchronicity thus goes beyond the synchronization of oscillators in positing a new kind of order in the natural world, schematized by Jung and Pauli (Fig. 2) in their book The Interpretation of Nature and of the Psyche [30]. Regular oscillator models fall far short of explaining synchronicities of this type, as Strogatz observed in his

$^1$ No reference is made in this paper to the use of archetypes in physical theory or other aspects of Jung’s philosophy.
A particularly important instance is the synchronization of matter and mind. In this view, mind is not slaved to the objective world, but tends to synchronize with it, based on limited exchange of information. Jung’s examples of synchronicity, and subjective perceptions of synchronicity generally, are often dismissed as the result of chance, but a minority opinion follows Pauli in asserting that a synchronistic order exists in the world alongside the causal one.

The point of this paper, reached through a review of the author’s and others’ previous work, is to show that the nonlinear dynamics paradigm of synchronization in networks of loosely coupled chaotic systems can realize the philosophical notion of synchronicity, or at least approach it much more closely than is possible with regular oscillators. The proposed realization is concrete in nature, without any need for dualism between the mental and material worlds. It is also different from the “dual-aspect monism” ascribed to Jung and Pauli themselves [4], in that material synchronization is put forward as an explanation of synchronous relationships in the mental realm, and between mind and matter.

We begin by showing, in the next section, that the simple introduction of a time delay in the coupling between the systems can transform a situation of complete synchronization to one of isolated “synchronicities”. In Section III, we review previous work on an application of synchronized chaos to “data assimilation” of observations of a “real” system into a computational model that is intended to synchronize with truth, analogously to the synchronization of matter and mind. “Meaningfulness” is naturally interpreted as internal coherence. A three-way relationship between two parts of a real system and a third system conceived as an observer is shown to satisfy the requirement for meaningfulness in synchronicities in Section IV.

The objective rational basis for synchronicity that is put forward in this paper suggests applications of the new organizational principle to processes in the brain and in the physical world. In Section V, we discuss implications of synchronized chaos for neural systems, in view of contemporary ideas about synchronization as a binding mechanism in perception and consciousness. In Section VI, we argue that synchronized chaos can support quantum nonlocality and the Bell correlations in a realist interpretation, where the difference with classical physics is less fundamental, but one may need to follow Einstein’s tradition in abandoning simple space-time geometry, and go further in this direction than did he. The concluding section speculates on remaining gaps between our objective realization of the
synchronicity principle and its original philosophical motivation.

II. HIGHLY INTERMITTENT SYNCHRONIZATION IN LOOSELY COUPLED CHAOTIC SYSTEMS

Extensive interest in synchronized chaotic systems was spurred by the work of Pecora and Carroll [44], who considered configurations such as the following combination of two Lorenz systems:

\[
\begin{align*}
\dot{X} &= \sigma(Y - X) \\
\dot{Y} &= \rho X - Y - XZ \\
\dot{Z} &= -\beta Z + XY \\
\dot{Y}_1 &= \rho X - Y_1 - XZ_1 \\
\dot{Z}_1 &= -\beta Z_1 + XY_1
\end{align*}
\]

which synchronizes rapidly: As \( t \to \infty \), \( Y_1(t) - Y(t) \to 0 \), \( Z_1(t) - Z(t) \to 0 \), as shown in Fig. 1. (Synchronization also occurs if the slave system is driven by the master \( Y \) variable instead of the \( X \) variable, but not if driven by the \( Z \) variable.) Various schemes to use chaos synchronization for cryptography were motivated by the thought that variables analogous to \( X \) in (1) could be used as carrier signals that would be difficult to distinguish from noise [45].

Synchronization can also occur with weaker forms of coupling than the complete replacement of one variable by its corresponding variable as in (1), but degrades below a threshold coupling strength. Typically, synchronization degrades via on-off intermittency [38], where bursts of desynchronization occur at irregular intervals, or as “generalized” synchronization [51], where a strict correspondence remains between the two systems, but that correspondence is given by a less tractable function than the identity. As shown schematically in Fig. 3, as differences between the two systems increases, the correspondence changes from the identity to a smooth function that approximates the identity, to one given by a function that is nowhere differentiable. The last case is in fact common [53].

While initial research on synchronized chaos was motivated by potential applications to secure communications, in applications to physical systems it is natural to consider forms of coupling that embody a time delay. If one extends chaos synchronization to the realm of
naturally occurring systems, the delay in transmission ought to be described in terms of the same physics that governs the evolution of the systems separately. To a first approximation let us assume that the time scale of the delay is the same as some intrinsic dynamical time scale of each system. Consider the following configuration of two Lorenz systems, coupled through an auxiliary variable \( S \) that introduces a delay:

\[
\begin{align*}
\dot{X} &= \sigma(Y - X) & \quad \dot{X}_1 &= \sigma(Y_1 - X_1) \\
\dot{Y} &= \rho(X - S) - Y - (X - S)Z & \dot{Y}_1 &= \rho(X_1 + S) - Y_1 - (X_1 + S)Z_1 \\
\dot{Z} &= -\beta Z + (X - S)Y & \dot{Z}_1 &= -\beta Z_1 + (X_1 + S)Y_1 \\
\dot{S} &= -\Gamma S + \Gamma(X - X_1)
\end{align*}
\]

The system (2) is a generalization of the Pecora-Carroll coupling scheme (1) to a case with bidirectional coupling and where each subsystem is partially driven and partially autonomous.

As \( \Gamma \to \infty \) in (2), with \( \dot{S} \) finite, \( S \to X - X_1 \). In this limit, the system reduces to a bidirectionally coupled version of (1), which indeed synchronizes. In the general case of the coupled system (2) with finite \( \Gamma \), the subsystems exchange information more slowly: if \( X \) and \( X_1 \) are slowly varying, then \( S \) asymptotes to \( X - X_1 \) over a time scale \( 1/\Gamma \). Thus \( \Gamma \) is an inverse time lag in the coupling dynamics.

Synchronization along trajectories of the system (2) is represented in Fig. 4 as the difference \( Z - Z_1 \) vs. time, for decreasing values of \( \Gamma \). For large \( \Gamma \), the case represented in Fig. 4a, the subsystems synchronize. As \( \Gamma \) is decreased in Figs. 4b-d, corresponding to increased time lag, increasingly frequent bursts of desynchronization are observed, until in Fig. 4d (uncoupled systems) no portion of the trajectory is synchronized. The bursting behavior can be understood as an instance of on-off intermittency [48],[38], the phenomenon that may occur when an invariant manifold containing an attractor loses stability, so that the attractor is no longer an attractor for the entire phase space, but is still effective in portions of the phase space. Trajectories then spend finite periods very close to the invariant manifold, interspersed with bursts away from it.

The case of a coupling time lag that is of the same order as the prescribed physical time scale in the simple Lorenz system corresponds to \( \Gamma = 1 \), with behavior as in Fig.
Although there is little trace of synchronization, the average instantaneous distance between the subsystems is less than it is in the uncoupled case. More interestingly, there is a very short period of nearly complete synchronization. In a very long integration, such "synchronicities", of moderately short duration, occur more commonly than they would by chance in unrelated systems, as seen in the histograms in Fig. 4e, showing total time in synchronicities of given duration for the two cases.

The system (2) is indeed analogous to one derived from a pair of geophysical fluid models coupled by standing waves in narrow ducts [11]. Auxiliary variables analogous to $S$ in (2) arise by first decomposing the field into a piece that satisfies the full nonlinear equations with homogeneous boundary conditions and a second piece that satisfies a linear system with matching boundary conditions in the region of the narrow ducts. The linear equations are solved using boundary Green’s functions that effect a time delay. The auxiliary variables are integrals of products of the boundary Green’s functions and differences of corresponding field variables from the two sides of the ducts. Intermittent synchronization of the two ODE systems implies correlations between large-scale weather patterns in the midlatitude regions of the Northern and Southern Hemispheres, respectively [11].

Small-world networks: One can consider a large array of chaotic oscillators of the type described above with synchronization among subsets or the entire array. In this context, the chaos synchronization phenomenon merges nicely with that of small world networks, or more generally, scale free networks in which the number of highly connected nodes decreases with the number of their connections according to a power law [55]. Randomly connected networks might be expected to synchronize more readily than regular networks that are connected only in local neighborhoods: the introduction of a few long-range connections can lead to a phase transition to long-range synchronization. [34],[5],[28],[66].

III. MACHINE PERCEPTION AS CHAOS SYNCHRONIZATION

The connection between synchronized chaos and mind-matter synchronicity is best illustrated by an application to meteorology. Computational models that predict weather include a feature not found in numerical solutions of simpler initial-value problems: As new data is provided by observational instruments, the models are continually re-initialized. This data assimilation procedure combines observations with the model’s prior prediction of the
current state - since neither observations nor model forecasts are completely reliable - so as to form an optimal estimate of reality at each instant in time. While similar problems exist in other fields, ranging from financial modelling to factory automation to the real-time modelling of biological or ecological systems, data assimilation methods are more developed in meteorology than in other fields.

Since the problem of data assimilation arises in any situation requiring a computational model of a parallel physical process to track that process as accurately as possible based on limited input, it is suggested here that the broadest view of data assimilation is that of machine perception by an artificially intelligent system. Like a data assimilation system, the human mind forms a model of reality that functions well, despite limited sensory input, and one would like to impart such an ability to the computational model.

The usual approach to data assimilation is to regard it as a tracking problem that can be solved using Kalman filtering or generalizations thereof. But clearly the goal of any data assimilation is to synchronize model with reality, i.e. to arrange for the former to converge to the latter over time. Thus the synchronously coupled systems of the previous section are re-interpreted as a “real” system and its model. In the system (1), for instance, we imagine that the world is a Lorenz system, that only the variable $X$ is observed, and that the observed values are passed to a perfect model.

The above philosophical considerations have motivated an attempt to recast data assimilation as a synchronization problem and thus to improve assimilation algorithms. It is first necessary to show that the synchronization phenomenon persists as the dynamical dimension of the model is increased to realistic values. Chaos synchronization in the sort of models given by systems of partial differential equations that are of interest in meteorology and other complex modelling situations has indeed been established. Pairs of 1D PDE systems of various types, coupled diffusively at discrete points in space and time, were shown to synchronize by Kocarev et al. [32].

Synchronization in geofluid models that are relevant to weather prediction was demonstrated by Duane and Tribbia [14],[15]. The models [62] are given in terms of the streamfunction $\psi(x,y,i,t)$ in a 2-layer ($i = 1, 2$) channel, contours of which are streamlines of atmospheric flow, as shown in Fig. 5, and a derived field variable, the potential vorticity $q_i \equiv f_0 + \beta y + \nabla^2 \psi_i + R_i^{-2} (\psi_1 - \psi_2)(-1)^i$, with constants as defined in the reference. The
dynamical equation for each model is

\[ \frac{Dq_i}{Dt} \equiv \frac{\partial q_i}{\partial t} + J(\psi, q_i) = F_i + D_i \]  

(3)

where the Jacobian \( J(\psi, \cdot) \equiv \frac{\partial \psi}{\partial x} \frac{\partial \cdot}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \cdot}{\partial x} = v_y \frac{\partial \cdot}{\partial y} + v_x \frac{\partial \cdot}{\partial x} \) gives the advective contribution to the Lagrangian derivative \( D/Dt \). The equation (3) states that potential vorticity is conserved on a moving parcel, except for forcing \( F_i \equiv \mu(q^* - q_i) \) and dissipation \( D_i \) as defined by Duane and Tribbia [15]. The forcing induces a relaxation to a jet-like background flow \( \psi^* \) (Fig. 5a,b) with \( q^* \equiv q(\psi^*) \), injecting energy into the system.

Two models of the form (3), \( Dq^A/Dt = F^A + D^A \) and \( Dq^B/Dt = F^B + D^B \) were coupled diffusively through a modified forcing term \( F_k^B = \mu_k^c[q_k^A - q_k^B] + \mu_k^{ext}[q_k^* - q_k^B] \), where the flow has been decomposed spectrally and the subscript \( k \) on each quantity indicates the wave number \( k \) spectral component. The two sets of coefficients \( \mu_k^c \) and \( \mu_k^{ext} \) were chosen to couple the two channels in some medium range of wavenumbers.

It was found that the two channels rapidly synchronize if only the medium scale modes are coupled (Fig. 5), starting from different initial flow patterns. For unidirectional coupling, the synchronization would effect assimilation of Fourier-space data from the \( A \) channel into the \( B \) channel. It has been shown analytically that optimal synchronization is equivalent to Kalman filtering when the dynamics change slowly in phase space, so that the same linear approximation is valid at each point in time for the real dynamical system and its model.

When the dynamics change rapidly, as in the vicinity of a regime transition, one must consider the full nonlinear equations and there are better synchronization strategies than Kalman filtering or the popular method of ensemble Kalman filtering [23]. The deficiencies of the standard methods, which are well known in such situations, are usually remedied by ad hoc corrections, such as “covariance inflation” [2]. In the synchronization view, such corrections can be derived from first principles [17]

\[ ^2 \text{The previous calculation of the covariance inflation factors [17] needs to be corrected to properly account for the discrete character of the actual assimilation cycle as contrasted with the continuous assimilation in the theoretical model.} \]
IV. INTERNAL SYNC VS. MIND-MATTER SYNC AND THE ROLE OF MEANING

In the context of data-assimilation/machine-perception, the role of synchronism is indeed reminiscent of Jung’s notion of synchronicity in the relationship between mind and the material world. But in the latter view, and in the popular culture surrounding the notion, the alleged relationships between events, mental or physical, are detected without close inspection and are “meaningful”. In this section, it is suggested that meaningfulness is realized naturally in terms of the internal coherence that is typically present in any system that synchronizes with an external system, and thus that the scientific view of synchronization may satisfy philosophers in this regard as well.

Prior use of the idealized geophysical model considered suggests how “meaning” might enter. The quasigeostrophic channel model was originally developed to represent the geophysical “index cycle”, in which the large-scale mid-latitude atmospheric circulation vacillates, at apparently random intervals, between two types of flow [62]. In the “blocked flow” regime, e.g. Fig. 6a, a large high-pressure center, typically over the Pacific or Atlantic, interrupts the normal flow of weather from west to east and causes a build-up of extreme conditions (droughts, floods, extreme temperatures) downstream. In the “zonal flow” regime, e.g. Fig. 6b, weather patterns progress normally. Synchronization of flow states, complete or partial, implies correlations between the regimes occupied by two coupled channel models at any given time. Such correlations, in the context of synchronization between reality and model, are indeed meaningful to meteorologists and to the residents of the regions downstream of any blocks. Synchronization of two highly simplified versions of the channel model has been used to predict correlations between blocking events in the Northern and Southern hemispheres [11],[12], and synchronization of two channel models has been used to infer conditions under which Atlantic and Pacific blocking events can be expected to anticorrrelate [14],[15].

To generalize from the geophysical models, we note that blocks are “coherent structures”, as commonly arise in a variety of nonlinear field theories. Such structures, of which solitons are perhaps the best known example, persist over a period of time because of a balance between nonlinear and dispersive effects. While no generally accepted definition of “coherent structure” has been articulated, one view of their fundamental nature can support
the proposed general connection with meaningfulness. For a structure to persist, the dif-
ferent degrees of freedom of the underlying field theory must continue to satisfy a fixed
relationship as they evolve separately. That behavior defines generalized synchronization,
the phenomenon in which two dynamical systems synchronize, but with a correspondence
between states given by a relationship other than the identity \[51\]. Coherent structures
are then characterized by internal generalized synchronization within a system. As state
variables that are generally synchronized with other state variables reveal additional infor-
mation, it is proposed that such relationships capture “meaningfulness” in the usual sense
of that term.

Meaningfulness is even more naturally defined as internal synchronization within mind.
A response to a given external stimulus by any “element” of mind is likely to be deemed
meaningful if there are synchronized, parallel responses of other mental elements.

It remains to show that internal synchronization is required or likely in each of a pair of
dynamical systems that exhibit synchronized chaos. The essential role of coherence in syn-
chronizing systems was previously highlighted by considering a pair of Hamiltonian systems,
for which complete synchronization is precluded because phase-space volumes of ensembles
of trajectories are preserved, by Liouville’s Theorem \[20\]. We consider a nonlinear scalar
field theory that gives rise to “oscillons” - coherent structures in the field that oscillate in
fixed, randomly placed locations - as do similar structures that were first noted in vibrating
piles of sand \[60\]. The expansion of the universe plays a role in the cosmological case that
is analogous to the role of frictional dissipation in the sandpiles, but the system is governed
by a time-dependent Hamiltonian, and Liouville’s Theorem still applies. A one-dimensional
model is given by the relativistic scalar field equation, with a nonlinear potential term, in
an expanding background geometry described by a Robertson-Walker metric with Hubble
constant \(H\). Using covariant derivatives for that metric in place of ordinary derivatives, one
obtains the field equation

\[
\frac{\partial^2 \phi}{\partial t^2} + H \frac{\partial \phi}{\partial t} - e^{-2Ht} \frac{\partial^2 \phi}{\partial x^2} + V'(\phi) = 0
\]

The scalar field exhibits oscillon behavior for some forms of the nonlinear potential \(V\)
(Fig. 7a), but not for others (Fig. 7b).

Where oscillons exist, a crude form of synchronized chaos is observed for a pair of loosely
coupled scalar field systems (a configuration that is introduced to study the synchronization
patterns, without physical motivation), as seen in Fig. 8. The fields do not synchronize, but the oscillons in the two systems tend to form in the same locations. For a potential that does not support oscillons, the positional coincidence is trivially absent, and there is no correlation between corresponding components of the underlying field. Synchronization in this case can only be interpreted in terms of coherent structures in the separate systems.

In a system as simple as the 3-variable Lorenz model, the hypothesis about the relationship between internal and external synchronization is also validated. In this case the relationship gives insight about which variables can be coupled to give synchronized chaos. Along the Lorenz attractor, the variables $X$ and $Y$ partially synchronize, resulting in the near-planar shape, while $Z$ is independent. Consistently with the internal-external synchronization hypothesis, either $X$ or $Y$, but not $Z$, can be coupled to the corresponding variable in an external system to cause the two systems to synchronize.

To summarize: The meaningfulness of a synchronization pattern (as philosophically required) is naturally defined in terms of internal synchronization, or coherent structures, involving some of the variables that synchronize externally. But external synchronization usually implies the existence of internal synchronization, and hence meaning.

V. SYNC AS AN ORGANIZATIONAL PRINCIPLE IN COMPUTATIONAL MODELING

If chaos synchronization provides a rational foundation for philosophical synchronicity, it should give deeper insight regarding apparent synchronicity in physical and psychological phenomena and underlying mechanisms. In the psychological realm, it has already begun to appear that synchronized oscillations play a key role. Synchronized firing of neurons has been introduced as a mechanism for grouping of different features belonging to the same physical object [63],[25],[52]. Recent debates over the physiological basis of consciousness have centered on the question of what groups or categories of neurons must fire in synchrony.

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3 The phases of the oscillons do not necessarily agree, so neither do we have an example of phase synchronization - the celebrated phenomenon [47, 50] in which a system that is chaotic can nevertheless be assigned a phase which matches that of a second system. Oscillon frequencies depend on their amplitudes, which may be different for a pair of oscillons whose positions correspond. (Additionally, it is not clear how one would define a phase for a multi-oscillon system that would capture information about their positions.)
in a mental process for that process to be a “conscious” one [33]. It was argued previously that patterns of synchronized firing of neurons provide a particularly natural and useful representation of objective grouping relationships, with chaotic intermittency allowing the system to escape locally optimal patterns in favor of global ones [21], following an early suggestion of Freeman’s [24]. The observed, highly intermittent synchronization of 40Hz neural spike trains might play just such a role.

The role of spike train synchronization in perceptual grouping has led to speculations about the role of synchronization in consciousness [63],[49],[55],[33], but here we suggest a relationship on a more naive basis: Consciousness can be framed as self-perception, and then placed on a similar footing as perception of the objective world. In this view, there must be semi-autonomous parts of a “conscious” mind that perceive one another. In the interpretation of Section III, these components of the mind synchronize with one another, or in alternative language, they perform “data assimilation” from one another, with a limited exchange of information. The scheme has actually been proposed, and is currently being investigated, for fusion of alternative computational models of the same objective process in a practical context [61].

Taking the proposed interpretation of consciousness seriously, again imagine that the world is a 3-variable Lorenz system, perceived by three different components of mind, also represented by Lorenz systems, but with different parameters. The three Lorenz systems also “self-perceive” each other. Three imperfect “model” Lorenz systems were generated by perturbing parameters in the differential equations for a given “real” Lorenz system and adding extra terms. The resulting suite is: 

\[
\begin{align*}
\dot{x}_i &= \sigma_i(y_i - z_i) + \sum_{j \neq i} C^{x}_{ij}(x_j - x_i) + K_x(x - x_i) \\
\dot{y}_i &= \rho x_i - y_i - x_i z_i + \mu_i + \sum_{j \neq i} C^{y}_{ij}(y_j - y_i) + K_y(y - y_i) \\
\dot{z}_i &= -\beta_i z_i + x_i y_i + \sum_{j \neq i} C^{z}_{ij}(z_j - z_i) + K_z(z - z_i)
\end{align*}
\]

where \((x, y, z)\) is the real Lorenz system and \((x_i, y_i, z_i)\) \(i = 1, 2, 3\) are the three models. An extra term \(\mu\) is present in the models but not in the real system. Because of the relatively small number of variables available in this toy system, all possible directional couplings among corresponding variables in the three Lorenz systems were considered, giving 18 connection coefficients \(C^{A}_{ij} \quad A = x, y, z \quad i, j = 1, 2, 3 \quad i \neq j\). The constants \(K_A \quad A = \)
are chosen arbitrarily so as to effect “data assimilation” from the “real” Lorenz system into the three coupled “model” systems. The configuration is schematized in Fig. 9.

The connections linking the three model systems were chosen using a general result on parameter adaptation in synchronously coupled systems with mismatched parameters that was proved rigorously in [19]: If two systems synchronize when their parameters match, then under some weak assumptions it is possible to prescribe a dynamical evolution law for general parameters in one of the systems so that the parameters of the two systems, as well as the states, will converge. In the present case the tunable parameters are taken to be the connection coefficients (not the parameters of the separate Lorenz systems), and they are tuned under the peculiar assumption that reality itself is a similar suite of connected Lorenz systems. The general result [19] gives the following adaptation rule for the couplings:

$$\dot{C}_{x_{i,j}} = a(x_j - x_i) \left( x - \frac{1}{3} \sum_k x_k \right) - \epsilon/(C_{x_{i,j}} - 100)^2 + \epsilon/(C_{x_{i,j}} + \delta)^2$$

with analogous equations for $\dot{C}_{y_{i,j}}$ and $\dot{C}_{z_{i,j}}$, where the adaptation rate $a$ is an arbitrary constant and the terms with coefficient $\epsilon$ dynamically constrain all couplings $C_{x_{i,j}}$ to remain in the range $(-\delta, 100)$ for some small number $\delta$. Without recourse to the formal result on parameter adaptation, the rule (6) has a simple interpretation: Time integrals of the first terms on the right hand side of each equation give correlations between truth-model synchronization error, $x - \frac{1}{3} \sum_k x_k$, and inter-model “nudging”, $x_j - x_i$. We indeed want to increase or decrease the inter-model nudging, for a given pair of corresponding variables, depending on the sign and magnitude of this correlation. (The learning algorithm we have described resembles a supervised version of Hebbian learning. In that scheme “cells that fire together wire together.” Here, corresponding model components “wire together” in a preferred direction, until they “fire” in concert with reality.) The procedure will produce a set of values for the connection coefficients that is at least locally optimal.

A simple case is one in which each of the three model systems contains the “correct” equation for only one of the three variables, and “incorrect” equations for the other two. The “real” system could then be formed approximately using large connections for the three correct equations, with other connections vanishing, and the peculiar assumption is strictly true if the large connections become infinite. Other combinations of model equations will also approximate reality.

In a numerical experiment (Fig. 10a), the couplings did not converge, but the coupled
suite of “models” did indeed synchronize with the “real” system, even with the adaptation process turned off half-way through the simulation so that the coupling coefficients $C_{i,j}^A$ subsequently held fixed values. The difference between corresponding variables in the “real” and coupled “model” systems was significantly less than the difference using the average outputs of the same suite of models, not coupled among themselves. (With the coupling turned on, the three models also synchronized among themselves nearly identically, so the average was nearly the same in that case as the output of any single model.) Further, without the model-model coupling, the output of the single model with the best equation for the given variable (in this case $z$, modeled best by system 1) differed even more from “reality” than the average output of the three models. Therefore, it is unlikely that any 	extit{ex post facto} weighting scheme applied to the three outputs would give results equalling those of the synchronized suite. Internal synchronization within the multi-model “mind” is essential. In a case where no model had the “correct” equation for any variable, results were only slightly worse (Fig. 10d).

The above scheme for fusion of imperfect computational/mental models only requires that the models come equipped with a procedure to assimilate new measurements from an objective process in real time, and hence from one another. The scheme has indeed been suggested for the combination of long-range climate projection models, which differ significantly among themselves in regard to the magnitude and regional characteristics of expected global warming [22]. (To project 21st century climate, the models are disconnected from reality after training, parameters are altered slightly to represent increased greenhouse gas levels, and one assesses changes in the overall shape of the attractor.) In this context, the previous results with Lorenz systems were thoroughly confirmed and extended using a learning method that minimizes synchronization error over finite-length trajectories, instead of the instantaneous error as above, to determine inter-model connections [61]. The scheme could also be applied to financial, physiological, or ecological models.

That the transition to synchronization among a suite of interconnected systems is sharper than the transition for a pair of systems is taken here to bolster the previous suggestions that synchronization plays a fundamental role in conscious mental processing. Here we imagine that the systems are neurons or collections of neurons. Note that the proposed role of synchronization is markedly different from Pauli’s view of mental phenomena as “something objectively psychical which cannot and should not be explained by material
causes” [4, 35]. Here mental phenomena are grounded in the material reality of neuronal systems, even if their dynamical properties are qualitatively different from those of the much higher dimensional physical world that they represent.

To describe ordinary mental phenomena, one needs a notion of synchronization at slower time scales and higher levels of organization. It remains to integrate a theory of higher-level synchronization with the known synchronization of 40Hz spike trains. It is certainly plausible that inter-scale interactions might allow synchronization at one time scale to rest on and/or support synchronization at other time scales. Inter-scale interactions played a similar role in the synchronization of a range of Fourier components of the same field in the synchronously coupled systems of partial differential equations considered in Section III. In a complex biological nervous system, with a steady stream of new input data, it is also very plausible that natural noise or chaos would result in very brief periods of widespread high-quality synchronization across the system (as in Fig. 4c), and possibly between the system and reality. It is conjectured that such “synchronicities” would appear subjectively as consciousness.

VI. SYNC IN QUANTUM THEORY

It is asserted that an “acausal connecting principle” applies not only to the relationship between mind and matter, or to relationships within mind, but also applies to matter itself. Turning to the realm of basic physics, the fundamental role of synchronism is most evident in the surprising long-distance correlations that characterize quantum phenomena. The Einstein-Podolsky-Rosen (EPR) phenomenon, viewed in light of Bell’s Theorem, implies that spatially separated physical systems with a common history continue to evolve as though connected with each other and with observers. Bell’s Theorem definitively asserts that observed correlations between two spatially separated spin-1/2 particles arising from the decay of a common spin-0 ancestor cannot be explained in terms of a causal relationship to the initial decay conditions alone. Such a relationship would imply that the binary-valued spins $A$ and $B$ are only functions of the orientations $a$ and $b$ of the respective measuring devices, and of some hidden variables represented collectively by $\lambda$, i.e. $A = A(a, \lambda)$, $B = B(b, \lambda)$. In general, $\lambda$ designates the state of the joint system at some initial time. One then defines the correlation $P$ between the two measured spins as a function of the two orientations.
\[ a \text{ and } b, \quad P(a, b) \equiv \int \rho(\lambda)A(a, \lambda)B(b, \lambda)d\lambda, \] where \( \rho(\lambda) \) is any function specifying a probability distribution of the hidden variables, i.e. \( \int \rho(\lambda)d\lambda = 1 \). But no matter how \( \lambda \) is defined or how its values are distributed, the correlations \( P \) are easily shown to satisfy a relationship that disagrees with standard quantum theory and with experiment [6], negating the assumed causal relationship. The picture of the quantum world that emerges from Bell’s Theorem is one in which everything is entangled with everything else, in a web of relationships similar to the one implied by the ubiquity of chaos synchronization. As with synchronized chaos, quantum entanglement can be used for cryptography, an analogy that was developed in prior work [13].

One may naturally seek an interpretation of EPR correlations in terms of synchronized chaotic oscillators, if one puts quantum theory on a non-local deterministic footing, as in Bohm’s interpretation [7],[8]. ’t Hooft’s more recent interpretation [58] involves pseudo-nonlocal connections, and an essential entanglement of the observer’s choice of \( a \) and \( b \) with the particle states [59]. The condition that connections of this sort not give rise to supraluminal transmission of information restricts the form of a deterministic theory. In this Section, we extend a previous speculation [16] that chaos synchronization can contribute to a realist interpretation of quantum theory.

Palmer [41] has suggested that the quantum world lives on a dynamically invariant fractal point set within the higher-dimensional phase space associated with the degrees of freedom that are naively thought to be independent. Membership in the invariant set is an uncomputable property, so theories can only be formulated in terms of the variables of the full phase space. Palmer’s invariant set is in fact a generalized synchronization “manifold” (the common but improper term, since the manifold is nowhere smooth), of the sort suggested by Fig. 3c. As discussed in prior work [16], generalized rather than identical synchronization is the fundamental relationship because EPR spins anti-correlate.

To bar supraluminal transmission of information, we rely on the mechanism proposed for synchronization-based cryptography: a signal provided by one variable of a chaotic system is difficult to distinguish from noise and is meaningful only when received by an identical copy of the system. However, it follows from Takens theorem that information can be extracted from such a signal if one considers a long enough time series. Longer time series are required to decode signals produced by more complex systems. For perfect security, one would need a chaotic system with an infinite-dimensional attractor.
Such a situation would arise most naturally in a multi-scale system (e.g. as proposed by Palmer[40]) requiring at least a system of partial differential equations, but something can be learned by considering a family of simpler systems of variable dimension, given by ordinary differential equations [13],[16]. It is known that two $N$-dimensional Generalized Rossler systems (GRS’s) (each equivalent to a Rossler system for $N = 3$) will synchronize for any $N$, no matter how large, when coupled via only one of the $N$ variables:

$$
\dot{x}_1^A = -x_2^A + \alpha x_1^A + x_1^B - x_1^A \\
\dot{x}_1^B = -x_2^B + \alpha x_1^B + x_1^A - x_1^B \\
\dot{x}_i^A = x_{i-1}^A - x_{i+1}^A \\
\dot{x}_i^B = x_{i-1}^B - x_{i+1}^B \\
\dot{x}_N^A = \epsilon + \beta x_N^A(x_{N-1}^A - d) \\
\dot{x}_N^B = \epsilon + \beta x_N^B(x_{N-1}^B - d)
$$

Each system has an attractor of dimension $\approx N - 1$, for $N$ greater than about 40, and a large number of positive Lyapunov exponents that increases with $N$. As $N \to \infty$, while the synchronization persists, the signal linking the two systems becomes impossible to distinguish from noise. It was shown previously [13] that an inequality analogous to Bell’s could be constructed by arbitrarily bisecting the phase space to define final states analogous to spin-up/spin-down, and using a GRS parameter as an analogue of measurement orientation. That inequality is in fact violated because of the connection between the systems, but a naive observer would expect it to hold because he is unable to distinguish the connecting signal from noise.

In Palmer’s view, there is no connecting signal because the world never leaves the “invariant set” (although the dissipative character of gravitational interactions is assumed to play a role cosmologically in dynamically constraining the universe to motion on the invariant set in the first place) [41]. Here we inquire as to the nature of the required “restoring force” if small perturbations transverse to the synchronization manifold are conceived as physical.

The GRS is a questionable model of reality because its “metric entropy,” the sum of the positive Lyapunov exponents, $\sum_{h_i>0} h_i$, is constant as $N \to \infty$, and that its largest Lyapunov exponent $h_{\text{max}} \to 0$ as $N \to \infty$. In other words, the higher the dimension, the less chaotic the system. Such behavior is suspect in a system intended to represent unpredictable quantum fluctuations. It is not known whether systems that are more chaotic than the GRS, but with attractors of arbitrarily high dimension, can be made to synchronize with
loose coupling, but it seems likely that such synchronization behavior would be restricted. PDE systems where chaos synchronization has been demonstrated have dissipative behavior that effectively gives a low-dimensional attractor that is part of an “inertial manifold” or approximate inertial manifold [15]. Without such low-dimensional behavior, synchronization of such systems by coupling a small number of variables would be more difficult. A deterministic theory underlying quantum behavior, such as that suggested by Palmer [40], would behave even more wildly than any PDE system.

Taking the GRS behavior as $N \to \infty$ to be generic, one must reconcile its increasingly mild character with the requirement that the nonlocal “signal” be perfectly masked through chaos. It was suggested previously [16] that the issue is resolved if the GRS is viewed as a spatially asymptotic description of an intrinsically faster dynamics in a highly curved space-time. For reference, recall that an object falling into a black hole is perceived by an observer at a distance from the hole as approaching the horizon with decreasing velocity, but never reaching it. If the physical system that the GRS describes lives in the vicinity of a micro-black hole or wormhole, the variables in the asymptotic description will be slowed, but the actual physical processes will be realistically violent, and can couple to each other through “signals” that are perfectly masked.

A. Nonlocality from Wormholes

A Planck-scale foam-like structure in space-time was posited by Hawking [26] in the context of a procedure to quantize classical general relativity where that structure contributes significantly to a sum over alternative Euclidean space-time geometries. Here we propose a role for microwormholes in long-range synchronization, without transmission of information, in ordinary Lorentzian space-time. Such a suggestion is consistent with theoretical arguments [9] and experimental evidence [1] for fundamental granularity in space-time structure. As explained in the Appendix, microwormholes may arise in a variant of general relativity defined by equations that are generally covariant but scale-dependent, and a weak divergence that arises from the recirculation of virtual quanta through wormholes may be avoided if the wormholes are sufficiently narrow.

The systems that must synchronize are defined on two-dimensional horizons at the mouths of the wormholes. It is consistent with the holographic principle [57],[56] that such 2D fields
capture the essential information about the full three-dimensional systems. Synchroniza-
tion of fields on 2D horizons is also consistent with suggestions that fields representing 2D
turbulence in fluids, but not 3D turbulence, can be made to synchronize [14].

If we stipulate, with Palmer, that the synchronization manifold is fundamental, because
the physical world never leaves it, then no wormholes are needed: We have two dynamical
systems defining an anticorrelated EPR pair, \( \dot{x} = F(x) \), and \( \dot{y} = G(y) \), with \( x \in \mathbb{R}^N \) and
\( y \in \mathbb{R}^N \). The dynamics are modified so as to couple the systems:

\[
\begin{align*}
\dot{x} &= \hat{F}(x, y) \\
\dot{y} &= \hat{G}(y, x)
\end{align*}
\]  

and there is some locally invertible function \( \Phi : \mathbb{R}^N \to \mathbb{R}^N \) such that \( ||\Phi(x) - y|| \to 0 \) as
\( t \to \infty \). Then the coupled dynamics are also defined by the two autonomous systems

\[
\begin{align*}
\dot{x} &= \hat{F}(x, \Phi(x)) \\
\dot{y} &= \hat{G}(y, \Phi^{-1}(y))
\end{align*}
\]  

without recourse to wormholes or any nonlocal connections, provided we know the badly
behaved function \( \Phi \) exactly. Otherwise, we rely on the narrow width of the wormholes to
prevent supraluminal transmission of matter or information. Diffraction effects preclude
communication, except in highly symmetrical situations, as in EPR, where constructive
interference might account for the needed nonlocal connections. The isolated character of
such quantum “synchronicities” follows from the rarity of the required symmetrical context.
Our wormholes are reminiscent of those in the original construction of Wheeler [65], who
suggested that lines of electric force are always closed if positive and negative charges are
thus connected at the microlevel.

That connections through narrow ducts can be sufficient to synchronize spatially extended
systems has already been demonstrated. Kocarev et al. [32] showed that pairs of PDE
systems of various types (Kuramoto-Sivishinsky, complex Ginsburg-Landau, etc.) could be
synchronized by pinning corresponding variables to one another at a discrete set of points,
at discrete instants of time. (The example of synchronizing two quasigeostrophic channel
models (Fig. 5) establishes essentially the same phenomenon for coupling formulated in
Fourier space.) Wormholes of zero length are expected to give synchronization of subsystems
on opposite sides in the same way. If the wormholes have finite length, then the resulting time
lags will lead to a system of the same form as (2) that gives intermittent synchronization.

The mediation of quantum interconnectedness by wormholes is perhaps the ultimate home
for the oft-proposed marriage [55] between synchronization dynamics and small-world (or
"scale-free") networks. The proposal might also realize the program, favored by a minority of physicists, of quantizing gravity by rooting quantum behavior in space-time geometry, rather than the reverse. The question is essentially whether the construction can reproduce the nonlocal piece of the "quantum potential" in Bohm’s interpretation [7] (the remaining piece corresponding to motion along the synchronization manifold), accounting for the origin of that piece geometrically.

VII. SUMMARY AND CONCLUDING REMARKS

In the foregoing sections, we have attempted to show that the synchronization of loosely coupled chaotic systems approaches the philosophical notion of highly intermittent, meaningful synchronicity more closely than commonly thought. Synchronized chaos is highly intermittent in a natural setting (Section II). As with philosophical synchronicity, it describes the relationship between the objective world and a perceiving mind (Section III). Central to our thesis is a relationship between internal synchronization within a system, and external synchronizability with another physical system or with a model. That relationship, which was described in Section IV, is in accord with common wisdom: An objective system with a high degree of internal synchronization is more easily perceived/understood, an internally coherent individual can more easily engage the world, etc.

What is not clear is that even with the isolated character and meaningfulness of synchronicities in coupled chaotic systems, the phenomenon reaches all the way to that of Jung and others, who discussed detailed coincidences between physical events and previous dreams, for instance. The attempt to put relationships of that kind on a rational footing may appear doomed. The mechanisms of deterministic chaos seem insufficient. One may dismiss such examples and consider only more restricted forms of synchronicity, or one may imagine that strange new physical principles are at work. The difficulty of ascribing the more extended notion of synchronicity to material reality may indeed have led to the dual-aspect monist conjecture [4] in which mind is elevated to the same level as matter, and both are aspects of an underlying domain that is neither mental nor material. In contrast, our view is decidedly materialist. Our endeavor might be compared to Marx’s attempt to ground Hegel’s dialectic in material reality, a transformation whose legitimacy has sometimes been questioned, notably by Bohm [43].
The question is perhaps sharpest in regard to consciousness and synchronization-based theories thereof. In Section V, it was argued that previous suggestions about the role of synchronization in the brain were supported by the possibility of highly intermittent synchronization among chaotic oscillators and by the possibility of synchronizing different complex models of the same objective process, giving rise to “self-perception”. But Penrose has given a well known argument that the reasoning abilities of conscious beings cannot arise from classical physics or algorithmic processes that describe such physics: For any algorithmic system of ascertaining truth, one can always articulate a true statement, of the sort constructed by Gödel, that such a being knows to be true, but whose truth cannot be established within the system [46]. Since synchronized chaos is still deterministic\textsuperscript{4}, the abilities of conscious beings must come from fundamentally different processes, which Penrose has suggested are quantum mechanical.

The discussion of quantum processes in Section VI was included because they seem to provide the deepest example of synchronicity - the quantum world appears to live on a generalized synchronization “manifold”. But if Penrose is correct, the converse statement can also be made: Synchronicity as manifest in human consciousness is also fundamentally quantum in origin. Correlations in neuronal firing or between neural subsystems can only give rise to consciousness, in this view, if quantum correlations are involved. Synchronicities between states of the mind and of the objective world must somehow follow. Perhaps such an enlarged notion could reach the popular concept, and the one of Jung and Pauli. In any case, it seems likely that the question of the proper interpretation of quantum phenomena on the one hand, and that of the origin of synchronism between mind and matter on the other, will be resolved jointly.

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\textsuperscript{4} If one considers a chaotic system given by differential equations for which infinite precision in initial conditions is needed to predict the outcome even qualitatively, as in Palmer’s earliest proposal [39], a typical basin of attraction for a given outcome is a “fat fractal”: The more precisely the initial conditions are known, the smaller is the probability of error in “guessing” the outcome. That is very unlike quantum indeterminacy.
Appendix A: On the possibility of microwormholes

As discussed in Section VI of the text, if space-time were permeated with micro-scale Wheelerian wormholes, that would be useless for time travel, a synchronistic order might nonetheless emerge: chaos synchronization might combine with small-world effects in the manner that has been suggested for more familiar applications. Here we discuss this possibility in light of two historical objections to wormholes.

A.1 Implications of the weak-energy condition in ordinary and higher-derivative gravity

While one might imagine that two Schwarzchild black hole solutions to Einstein’s equations could be joined to form a wormhole, solutions of this type are not traversible [36]. The possibility of traversible wormhole solutions is limited by the *weak energy condition*. That condition states that for any null geodesic, say one parameterized by \( \zeta \), with tangent vectors \( k^a = dx^a/d\zeta \), an averaged energy along the geodesic must be positive:

\[
\int_0^\infty T_{\alpha\beta}k^\alpha k^\beta > 0 \tag{A1}
\]

where \( T_{\alpha\beta} \) is the stress-energy tensor. Traversable wormholes can exist only if A1) is violated for some null geodesics passing through the wormhole, implying the existence of “exotic matter” with negative energy density in the “rest frame” of a light beam described by the null geodesic. The negative energy density is required, in one sense, to hold the wormhole open.

Quantum fluctuations in the vacuum can violate the weak energy condition [37],[10]. But the problem might be avoided at the classical level, as desired if quantum theory is not to be presumed, if a larger class of generally covariant theories are considered. Terms containing higher derivatives of the metric can indeed be added to Einstein’s equations, with effects that are negligible on all but the smallest scales [64],[54]. The situation is analogous to that of the Navier-Stokes equation in fluid dynamics: While the terms involving the co-moving derivative follow simply from Newton’s first law, the dissipative terms are ad hoc and can take many forms. General relativity can likewise be extended to theories of the form:

\[
R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + g_{\mu\nu}\Lambda + \sum_{n>2} c_n L^{n-2} R^{(n)}_{\mu\nu} = 8\pi T_{\mu\nu} \tag{A2}
\]
where $R_{\mu\nu}^{(n)}$ is a quantity involving a total of $n$ derivatives of the metric, $L$ is a fundamental length scale, the $c_n$ are dimensionless constants, and we have included a cosmological constant $\Lambda$ for full generality. If $L = L_P$, the Planck length, then the new terms in the extended theory (A2) are negligible on macroscopic scales. They only need be considered if curvature is significant at the Planck length scale. Any metric that solves the ordinary Einstein equations after the substitution $T_{\mu\nu} \to T_{\mu\nu} - (1/8\pi) \sum_{n>2} c_n L^{n-2} R_{\mu\nu}^{(n)}$ solves (A2) for given $T_{\mu\nu}$. It is plausible that the modified stress-energy tensor $T_{\mu\nu} - (1/8\pi) \sum_{n>2} c_n L^{n-2} R_{\mu\nu}^{(n)}$ can be made to violate the weak energy condition if the signs of the constants $c_n$ are chosen appropriately, and thus that a traversible micro-wormhole solution is possible.

A.2 Vacuum recirculation effects for narrow wormholes

The paradoxes that one normally associates with closed time-like curves, as would pass through a wormhole, have a quantum counterpart: Repeated passage of a virtual particle through a wormhole may lead to a divergence in the stress-energy tensor $T_{\mu\nu}$. The derivation of this controversial result is as follows: For each passage of a virtual particle through the wormhole, the contribution to the two-point function $\langle \Psi | \hat{\phi}(x)\hat{\phi}(x') + \hat{\phi}(x')\hat{\phi}(x) | \Psi \rangle$ from a trajectory that contains that passage is attenuated by a factor $b/D$, where $b$ is the wormhole width, and $D$ is the spatial length of a geodesic through the wormhole, as measured in the frame of an “observer” traveling along the geodesic from the vicinity of $x$ and $x'$ through the wormhole once and back to the same vicinity. Here, $x$ and $x'$ are nearby points in spacetime, $|\Psi\rangle$ is the quantum state, and $\hat{\phi}$ is the field operator associated with the field $\phi$. The contribution to the two-point function is found to behave as $(b/D)^k \times 1/\sigma$, where $\sigma$ is $1/2$ the square of the proper distance between $x$ and $x'$ along the geodesic connecting them through the wormhole, and the power $k$ depends on the number of times that the trajectory traverses the wormhole. (Contributions from trajectories that traverse the wormhole only once dominate.) One finds $\sigma \sim D\Delta t$, where $\Delta t$ is the proper time between $x$ and the nearest null geodesic that passes through the wormhole. As $x' \to x$, the contribution diverges if $x$ can be joined to itself by a null geodesic that passes through the wormhole. The stress-energy tensor can be expressed in terms of the two-point function [31] and also diverges as $\sigma \to 0$ or $\Delta t \to 0$. Specifically, one finds $T_{\mu\nu} \sim (b/D)^k \times 1/D(\Delta t)^3$ in natural units, or in
dimensional units,

\[ T_{\mu\nu} \sim (b/D)^k \times L_P/D \times m_P/(\Delta t)^3. \quad \text{(A3)} \]

Kim and Thorne [31] argued that the divergence, which is small because of the “diffraction” factor \( b/D \), probably disappears in the proper quantum theory of gravity, allowing wormholes to remain. Quantization of the gravitational field in that theory would be effective on scales of \( L_P \), the Planck length, so we only need consider the magnitude of \( T_{\mu\nu} \) for \( \sigma \geq L_P \). At these scales, referring to (A3), \( T_{\mu\nu} \leq L_P/D \) in natural units of \( m_P/L_P^3 \), giving energy densities that are far too weak to destroy the wormhole, or have other noticeable effects, for macroscopic \( D \).

Hawking [27], in support of his “chrononology protection conjecture”, provided a counter-argument asserting that quantum gravity effects would only enter on much smaller scales, corresponding to the Planck length in the rest-frame of an “observer” travelling on one of the geodesics through the wormhole. The values attained by \( T_{\mu\nu} \) on scales larger than Hawking’s reduced length scale would still cause collapse of the wormhole, the instant that recirculation becomes possible.

Let us assume that the predictions of standard quantum theory in curved spacetime survive in whatever deterministic theory might underlie quantum mechanics. Here, we note that there is an additional mechanism that might cut off the recirculation divergence for wormholes of very narrow width. Virtual particles of arbitrarily high energy cannot traverse the wormhole. High-energy virtual particles would reverse the effect of the exotic matter or of the higher-derivative terms, so the existence of the wormhole would again be precluded by the weak energy condition. The contribution to the energy flux from the virtual particles is \( T^{0i} = \frac{4}{5\pi} \int d\omega n(\omega)\hbar \omega \), where \( n(\omega) \) is the number density of quanta at frequency \( \omega \). (As in the Weizsacker -Williams approximation [29], the quanta are assumed not to elongate in the wormhole.) At detailed resolution in frequency-space, \( n(\omega) = \sum_{i} n_i \delta(\omega - \omega_i) \), where \( \omega_i \) is a discrete set of frequencies and \( \{n_i\} \) is a set of positive integers. There is a problem from the weak energy condition if any \( \omega_i > \omega_{\text{cutoff}} \) (with \( n_i \geq 1 \), for \( \omega_{\text{cutoff}} \) sufficiently large as to cancel the negative-energy contributions to \( T^{00} \). In a path integral, taken both over particle trajectories and over geometries, one need only consider histories in which more energetic particles either collapse the wormhole or are reflected and do not traverse it. In contrast, for wormholes of macroscopic width, histories must be included in the path integral for which the energies of recirculating virtual quanta outside the wormhole are anomalously large.
(treating the geometry itself classically). The cutoff in the former case implies that the term $1/\sigma$ in the two-point function is replaced by a term like $\int_{\omega < \omega_{\text{cutoff}}} d^4k \exp[ik \cdot (x - x')] / k^2$ which does not diverge.

Thus for sufficiently narrow wormholes, the original position of Kim and Thorne that the vacuum recirculation divergence is damped might be correct. Further, although finite wormhole lifetime would be required to mediate long-range synchronization, highly intermittent wormhole behavior may be enough, in accordance with previous findings [32] for other types of PDE’s. In the absence of a detailed Planck-scale theory, the question remains open.


FIG. 1: The trajectories of the synchronously coupled Lorenz systems in the Pecora-Carroll complete replacement scheme (1) rapidly converge (a). Differences between corresponding variables approach zero (b).

FIG. 2: Diagram constructed by Carl Jung, later modified by Wolfgang Pauli, to suggest relationships based on synchronicity as an “acausal connecting principle”, existing alongside causal relationships [30].

FIG. 3: Transition from identical to generalized synchronization, illustrated by the relationship between a pair of corresponding variables $x$ and $x'$: Projection of the synchronization manifold onto the $(x, x')$ plane are shown for (a) identical synchronization, (b) generalized synchronization with near-identical correspondence, (c) generalized synchronization with a correspondence function that is not smooth.

FIG. 4: The difference between the simultaneous states of two Lorenz systems with time-lagged coupling (2), with $\sigma = 10., \rho = 28.,$ and $\beta = 8/3$, represented by $Z(t) - Z_1(t)$ vs. $t$ for various values of the inverse time lag $\Gamma$ illustrating complete synchronization (a), intermittent or “on-off” synchronization (b), partial synchronization (c), and de-coupled systems (d). Average euclidean distance $\langle D \rangle$ between the states of the two systems in $X, Y, Z$-space is also shown. A histogram of the lengths of periods of “synchronicity”, such as the one indicated by the arrow in (c), is shown in (e) for the time-delayed coupling case (solid line) and a case of two unrelated Lorenz trajectories (dashed line), where synchronicity intervals are periods during which $|Z(t) - Z_1(t)| < 5$.

FIG. 5: Streamfunction (in units of $1.48 \times 10^9 m^2 s^{-1}$) describing the forcing $\psi^* (a,b)$, and the evolving flow $\psi (c-f)$, in a parallel channel model with coupling of medium scale modes for which $|k_x| > k_x0 = 3 \text{ or } |k_y| > k_y0 = 2$, and $|k| \leq 15$, for the indicated numbers $n$ of time steps in a numerical integration (generalizing to bidirectional coupling, for convenience). Parameters are as described previously [15]. An average streamfunction for the two vertical layers $i = 1, 2$ is shown. Synchronization occurs by the last time shown (e,f), despite differing initial conditions.
FIG. 6: Streamfunction (in units of $1.48 \times 10^9 m^2 s^{-1}$) describing a typical blocked flow state (a) and a typical zonal flow state (b) in the two-layer quasigeostrophic channel model. An average streamfunction for the two vertical layers $i = 1, 2$ is shown.

FIG. 7: Energy density $\rho = (1/2)e^{-Ht}(\phi_x)^2 + (1/2)e^{Ht}(\phi_t)^2 + e^{Ht}V(\phi)$, vs. position $x$ for a numerical simulation of the scalar field equation (4) with the potential $V(\phi) = (1/2)\phi^2 - (1/4)\phi^4 + (1/6)\phi^6$, suggesting localized oscillons (a), and a simulation of the same equation, but with a different potential $V(\phi) = (1/2)\phi^2 + (1/4)\phi^4 + (1/6)\phi^6$, for which oscillons do not occur (b).

FIG. 8: The local energy density $\rho$ vs. $x$ for two simulations of the scalar field equation (4), coupled to one another only through modes of wavenumber $k \leq 64$, where modes up to $k_{\text{max}} = 2^{14}$ are realized numerically. ($\rho$ for the second system (dashed line) is also shown reflected across the x-axis for ease in comparison.) The coincidence of oscillon positions is apparent.

FIG. 9: “Model” Lorenz systems are linked to each other, generally in both directions, and to “reality” in one direction. Separate links between models, with distinct values of the connection coefficients $C_{ij}^l$, are introduced for different variables and for each direction of possible influence.

FIG. 10: Difference $z_m - z$ between “model” and “real” $z$ vs. time for a Lorenz system with $\rho = 28$, $\beta = 8/3$, $\sigma = 10.0$ and a suite of models with $\rho_{1,2,3} = \rho$, $\beta_1 = \beta$, $\sigma_1 = 15.0$, $\mu_1 = 30.0$, $\beta_2 = 1.0$, $\sigma_2 = \sigma$, $\mu_2 = -30.0$, $\beta_3 = 4.0$, $\sigma_3 = 5.0$, $\mu_3 = 0$. The synchronization error is shown for a) the average of the coupled suite $z_m = (z_1 + z_2 + z_3)/3$ with couplings $C_{ij}^A$ adapted according to (6) for $0 < t < 500$ and held constant for $500 < t < 1000$; b) the same average $z_m$ but with all $C_{ij}^A = 0$; c) $z_m = z_1$, the output of the model with the best $z$ equation, with $C_{ij}^A = 0$; d) as in a) but with $\beta_1 = 7/3$, $\sigma_2 = 13.0$, and $\mu_3 = 8.0$, so that no equation in any model is “correct”. (Analogous comparisons for $x$ and $y$ give similar conclusions.)
FIG. 1:
FIG. 2:
FIG. 3:
FIG. 4:
FIG. 5:
a) CONTOUR FROM -0.075 TO 0.075 BY 0.005

b) CONTOUR FROM -0.11 TO 0.11 BY 0.01

FIG. 6:
FIG. 7:
FIG. 8:
FIG. 9:
FIG. 10: