Perfect models and perfect/noisy observations

Imperfect models

Short-term forecast with imperfect models

Long-term (climate) forecast with imperfect models

Conclusions and discussions

Forecasting with imperfect models

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Potsdam, 20 November 2012
The ability to make good forecasts is limited both by uncertainty in the state of the models and by imperfections in the model (Bjerknes, 1911).

For perfect model scenario and perfect (noise free) observations: from a single scalar measurement of the system, one can (generically) obtain the true state from sufficiently many (but finite in number) observations (Takens 1981, but also Aeyels, 1981).
Perfect models and noisy observations

There are (at least) two fundamental limits when dealing with chaotic systems:

- Sensitivity to initial conditions prevents long term forecasting of the future state.
- Given noisy observations (of arbitrary duration) and a perfect model of the system, sensitivity to initial conditions also limits the ability to identify the true (current) state (Judd and Smith, 2001).
Consequences:

- There is a set of states indistinguishable from the true state.
- An accurate forecast must be based on a probability density on the indistinguishable states.
- Various procedures called state estimation, filtering, tracking, and data assimilation are, from a mathematical point of view, aiming to solve the same problem: *state identification*, perhaps with some differences in emphasis and terminology.
Imperfect models

Perfect model scenario is a fiction: in practice all models are imperfect (Judd and Smith, 2004). Two types of imperfect models:

- Structurally incorrect model: the system dynamics are not known in detail or cannot be expressed in terms of known mathematical functions.

- Ignored-sub space model: the system has a component of its dynamics that is unknown, unobservable, or not included in the model.
Examples of toy models for imperfect model scenario

Pair of structurally different systems (truth, model) for short- or long-term prediction

- Lorenz 63 driven by Lorenz 63, Lorenz 63
- Lorenz 96 class III, Lorenz 96 class II
Lorenz 63 system driven by another Lorenz 63 system

Observable (1) and unobservable part (2)

\[
\begin{align*}
\dot{x}_1 & = \sigma(y_1 - x_1) + \epsilon z_2, \\
\dot{y}_1 & = x_1(\rho - z_1), \\
\dot{z}_1 & = x_1 y_1 - \beta z_1 + \delta(x_2 + \eta), \\
\dot{x}_2 & = \sigma(y_2 - x_2), \\
\dot{y}_2 & = x_2(\rho - z_2), \\
\dot{z}_2 & = x_2 y_2 - \beta z_2.
\end{align*}
\]

- Standard parameter values $\rho = 28$, $\sigma = 10$ and $\beta = 8/3$.
- Driving intensities $\delta = 5$ and $\epsilon = 1$; drift $\eta = 2$.
Models of driven Lorenz 63 system

\[
\begin{align*}
\dot{x} &= \sigma(y - x) + \alpha, \\
\dot{y} &= x(\rho - z), \\
\dot{z} &= xy - \beta z + \gamma
\end{align*}
\]

- Model 1 - ordinary Lorenz 63 system ($\alpha = 0; \gamma = 0$)
- Model 2 - Weighted SUMO of Lorenz 63 system with parameterized forcing terms ($\alpha = 0$ or $\gamma = 0$)
- Model 3 - Learning the model’s tendencies as quadratic polynomials of $x$, $y$, and $z$;
- Model 4 - Leith method - statistics-based improvement of the ordinary Lorenz system with linear terms
Anomaly correlation for models of driven Lorenz 63 system

SNR = 25

- Leith
- Lorenz 63
- Learned 63
- Weighted sumo

Time units

AC
What is climate?

- Leith (Nature, 1978): Climate at any place is commonly defined as being the average weather there. The more precise definition includes all statistical properties of the global atmosphere and the underlying land, sea, and ice surfaces.
- Climate is the probability density function of the weather attractor.
- For practical reasons climate consists of averages and (co)variances of some quantities calculated as time averages for some period.
Toy model for climatological tests - Lorenz 63 with linearly increasing parameter $\rho$

\[
\begin{align*}
\dot{x} &= \sigma [y - x], \\
\dot{y} &= x [\rho(t) - z], \\
\dot{z} &= xy - \beta z, \\
\rho(t) &= \rho_0 + \alpha t
\end{align*}
\]

- As climatological variable is considered the time-average of the variable $z$ for some period
- Ensemble of models - climate is the ensemble average; as measure of climate variability is the variance of the ensemble
Variance of ensemble of Lorenz 63 systems with linearly increasing parameter $\rho$
Lorenz 96 system with linearly increasing forcing - model (class II)

\[ \dot{X}_n = [X, X]_{K,n} - X_n + F_0(1 + t/T), \]

\[ [X, Y]_{K,n} = \sum_{j=-J}^{J} \sum_{i=-J}^{J} \frac{-X_{n-2K-i}Y_{n-K-j} + X_{n-K+j-i}Y_{n+K+j}}{K^2} \]

- One dimensional scalar variable \( X \) over one latitude circle taken at \( N \) points
- Forcing term doubles its value in period \( T = 100 \) years
- The (ensemble of) models should predict the annual average of the variable \( \langle Z \rangle_{\text{annual}} \) - next page
Lorenz 96 system with linearly increasing forcing - truth (class III)

\[
\dot{Z}_n = [X, X]_{K,n} + b^2 [Y, Y]_{1,n} + c [Y, X]_{1,n} - X_n - b Y_n + F_0 (1 + t/T)
\]

- Large-scale variable \( X_n = \sum_{i=-l}^{l} (\alpha - \beta |i|) Z_{n+i} \)
- Small-scale variable \( Y_n = Z_n - X_n \)
- \( b \) and \( c \) – parameters; filter length \( l \) and filter parameters \( \alpha \) and \( \beta \)

\[
\alpha = \frac{3l^2 + 3}{2l^3 + 4l}, \quad \beta = \frac{2l^2 + 1}{l^4 + 2l^2}
\]
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Lorenz 96 system with linearly increasing forcing
Improved climate projection of Lorenz 96 system

Year
Global yearly average of x (z) and ensemble spread
N=480

2.6
2.8
3
3.2
3.4
3.6
3.8
4
4.2
4.4

Truth
N480 ens mean dc = 1.2
N480 ens spread
N480 ens mean dc=1.0
N480 ens spread

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Conclusions

- All models are imperfect.
- Weather prediction is a state identification problem.
- Climate forecast requires ensembles of multi-decadal simulations to assess both (chaotic) climate variability and model response.